

by Intense Solar Radiation," *Journal of Solar Energy Engineering*, Vol. 107, No. 1, 1985, pp. 29–34.

<sup>3</sup>Siegel, R., "Internal Radiation Effects in Zirconia Thermal Barrier Coatings," *Journal of Thermophysics and Heat Transfer*, Vol. 10, No. 4, 1996, pp. 707–709.

<sup>4</sup>Siegel, R., "Two-Flux Green's Function Analysis for Transient Spectral Radiation in a Composite," *Journal of Thermophysics and Heat Transfer*, Vol. 10, No. 4, 1996, pp. 681–688.

<sup>5</sup>Siegel, R., "Two-Flux and Green's Function Method for Transient Radiative Transfer in a Semitransparent Layer," *Proceedings of the 1st International Symposium on Radiative Heat Transfer* (Kuşadasi, Turkey), *Radiative Transfer—1*, Begell House, New York, 1996, pp. 473–487.

<sup>6</sup>Makino, T., Kunitomo, T., Sakai, I., and Kinoshita, H., "Thermal Radiative Properties of Ceramic Materials," *Transactions of the Japan Society of Mechanical Engineers*, Vol. 50, No. 452, 1984, pp. 1045–1052.

<sup>7</sup>Siegel, R., and Howell, J. R., *Thermal Radiation Heat Transfer*, 3rd ed., Hemisphere, Washington, DC, 1992, pp. 23, 33.

## Turbulent Flow and Heat Transfer in Rotating Different Aspect Ratio Channels

Sandip Dutta\*

University of South Carolina,  
Columbia, South Carolina 29208

and

Malcolm J. Andrews† and Je-Chin Han†  
Texas A&M University,  
College Station, Texas 77843-3123

### Introduction

A STUDY of heat transfer and turbulent flow in rotor blade coolant passages of different practically important aspect ratios is presented. The difficulty of collecting such data in the laboratory has led us to perform numerical experiments. Five different aspect ratios of the coolant passage are selected to cover different regions of a turbine blade. The leading edge of a blade has more space to accommodate a low aspect ratio coolant channel and the narrow trailing edge can have a high aspect ratio coolant passage.

Most earlier numerical work about rotating channels used either simple flow models (some of them inviscid models) or parabolic methods for unheated channels.<sup>1</sup> Recent work on heated channels by Prakash and Zerkle<sup>2</sup> and Tekriwal<sup>3</sup> included thermal buoyancy effects in the momentum and predicted heat transfer results with a high Reynolds number  $k$ - $\epsilon$  model obtaining reasonable qualitative agreement with experimental profiles of local Nusselt numbers.

### Mathematical Model and Conditions

A two-equation turbulence model with rotation-modified turbulence terms<sup>1</sup> are used. We have used a Boussinesq approximation for our analysis in which the density is constant except in the rotational buoyancy terms. Since flow Mach number is less than 0.1, compressible effects in the energy equation are neglected.

The rotational buoyancy and Coriolis-generated turbulence production terms in the  $k$ - $\epsilon$  transport equations,<sup>1</sup>  $P_b$  and  $P_c$ , are taken as

$$P_b = \frac{\mu_t}{Pr_t} \beta \Omega^2 r_z \frac{\partial T}{\partial z}, \quad P_c = 9\Omega \mu_t \frac{\partial w}{\partial x} \quad (1)$$

The buoyancy production term  $P_b$  arises from a Boussinesq approximation of the velocity-temperature cross correlation.<sup>4</sup> The Coriolis-modified term  $P_c$  is from Howard et al.<sup>5</sup> In general,  $P_c$  is positive near the trailing wall and negative near the leading wall. A positive  $P_c$  increases turbulence and a negative  $P_c$  suppresses turbulence.

The channel hydraulic diameter  $D$  is 6 mm for all aspect ratio channels. The aspect ratios, as shown in Fig. 1, are AR = 1:4, 1:2, 1:1 (square), 2:1, and 4:1. The mean rotating radius  $R$  is  $50D$ . The coolant is air at 30 atm. The inlet coolant temperature is 900 K and the surrounding heated surfaces are at 1200 K in the heated test section. The density ratio ( $DR = 1 - \rho_w/\rho_{in} = 1 - T_{in}/T_w$ ) obtained is  $DR = 0.25$ . The channel is rotated at 10,000 rpm. The Reynolds number ( $Re = \rho w_0 D / \mu$ , based on the  $D$  and inlet conditions) is varied from  $Re = 4 \times 10^3$  to  $1.5 \times 10^4$  to give rotation numbers ( $Ro = \Omega D / w_0$ ) of  $Ro = 0.3, 0.4, 0.5, 0.6$ , and  $0.7$ . Other nondimensional parameters used are rotational Grashof number ( $Gr = -\rho^2 \Omega^2 R \beta (T_w - T_{in}) D^3 / \mu^2$ ), Nusselt number ( $Nu = hD / K_{air}$ ), and  $Nu$  for fully developed turbulent pipe flow ( $Nu_0 = 0.023 Re^{0.8} Pr^{0.4}$ ). In the previous definitions,  $\Omega$  is the rotation speed,  $h$  is the heat transfer coefficient, and  $w_0$  is the mean channel flow velocity.

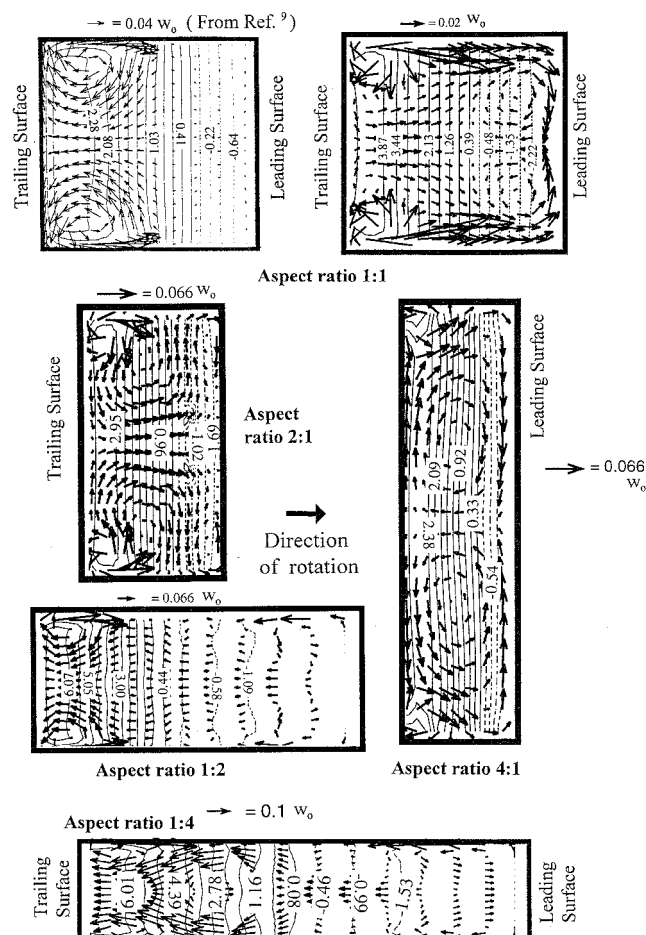


Fig. 1 Secondary flow vectors and axial flow contours ( $w/w_0$ ) in different aspect ratio channels.  $Re = 2.5 \times 10^4$ ,  $Ro = 0.5$ , and  $z_0/D = 10$ .

Received May 6, 1996; revision received Jan. 6, 1997; accepted for publication Jan. 7, 1997. Copyright © 1997 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved.

\*Assistant Professor, Department of Mechanical Engineering.

†Department of Mechanical Engineering.

## Results and Discussion

Figure 2 shows the effect of rotation number on Nusselt number ratio at selected axial locations. Data of Wagner et al.,<sup>6</sup> Han et al.,<sup>7</sup> and Guidez<sup>8</sup> are included for comparison. Wagner et al.<sup>6</sup> and Han et al.<sup>7</sup> used square rotating channels, and Guidez<sup>8</sup> used a 2:1 aspect ratio rectangular channel. The axial locations of Wagner et al.<sup>6</sup> are  $z_0/D = 4.7$  and  $8.5$ , and for Han et al.<sup>7</sup> the locations are  $z_0/D = 5$  and  $9$ . Whereas the axial location of Guidez's<sup>8</sup> data is  $z_0/D = 7.4$ . Results show that the trend in the existing experimental data may be used to estimate the coolant channel performance in the real operating condition. Results show that high aspect ratio channels show less effects of rotation on the heat transfer pattern from the leading and trailing surfaces. Whereas, the low aspect ratio channels show a wider difference between the Nusselt number ratios from trailing and leading surfaces. The trailing wall of 1:2 aspect ratio channel shows the highest Nusselt number ratios. However, the variations of this surface Nusselt number ratio is not smooth with rotation number. The trailing surface Nusselt number ratio is in general higher than the leading surface Nusselt number ratio. Coriolis force shifts the axial momentum from the leading to the trailing side and that shift increases the axial flow velocity near the trailing wall. Moreover, rotation redistributes the turbulence favoring heat transfer enhancement from the trailing wall.

Figure 1 compares the secondary flow vectors of square and other aspect ratio channels at  $z_0/D = 10$ . Predictions of Dutta et al.<sup>9</sup> are compared with the present analysis. Dutta et al.<sup>9</sup> predicted the flow conditions of Wagner et al.<sup>6</sup> ( $Re = 2.5 \times$

$10^4$ ,  $Ro = 0.24$ ,  $DR = 0.22$ ,  $z_0/D = 9$ , and  $Gr/Re = 9 \times 10^3$ ). The prediction of Dutta et al.<sup>9</sup> was based on coolant air at 10 atm, at an inlet temperature of 300 K, and the rotation speed was 575 rpm. The present analysis is for simulated engine conditions with a higher rotation speed of 10,000 rpm, the coolant is at 30 atm, and the inlet temperature is 900 K. The vector plot for  $z_0/D = 10$  of the present analysis is different from the flow structure of Dutta et al.<sup>9</sup> and the secondary flow is from trailing to the leading surface, which is opposite to the direction of the Coriolis force. This reverse secondary flow vortex indicates that the buoyancy effects (proportional to  $\Omega^2$ ) are stronger than the Coriolis effect (proportional to  $\Omega$ ) in this analysis. The contours in the plots are for the axial velocity. The plots show a stratified axial flow distribution in the channel with positive radial outward flow velocity near trailing surface and reversed radially inward separated flow near the leading surface.

Note that for this study, the direction of the secondary flow does not have a significant impact on the heat transfer. Figure 2 shows that the trailing side heat transfer coefficients are higher than the leading side heat transfer coefficients at all locations. Since the magnitude of the crossflow is less than 5% of the main flow in the core, the effect of secondary flow direction is not prominent in heat transfer coefficients. The Nusselt number ratio is more dependent on the increased bulk flow and turbulence enhancement near the trailing side.

## References

- Dutta, S., Andrews, M. J., and Han, J. C., "Prediction of Turbulent Flow and Heat Transfer in Rotating Square and Rectangular Smooth Channels," American Society of Mechanical Engineers, 96-GT-234, June 1996.
- Prakash, C., and Zerkle, R., "Prediction of Turbulent Flow and Heat Transfer in a Radially Rotating Square Duct," *Journal of Turbomachinery*, Vol. 114, Oct. 1992, pp. 835–846.
- Tekriwal, P., "Heat Transfer Predictions with Extended  $k-\epsilon$  Turbulence Model in Radial Cooling Channels Rotating in Orthogonal Mode," *Journal of Heat Transfer*, Vol. 116, May 1994, pp. 369–380.
- Hossain, M. S., and Rodi, W., "A Turbulence Model for Buoyant Flows and Its Application to Vertical Buoyant Jets," *Turbulent Buoyant Jets and Plumes*, edited by W. Rodi, Pergamon, New York, 1982, pp. 143–146.
- Howard, J. H. G., Patankar, S. V., and Bordyniuk, R. M., "Flow Prediction in Rotating Ducts Using Coriolis-Modified Turbulence Models," *Journal of Fluids Engineering*, Vol. 102, Dec. 1980, pp. 456–461.
- Wagner, J. H., Johnson, B. V., and Hajek, T. J., "Heat Transfer in Rotating Passages with Smooth Walls and Radial Outward Flow," *Journal of Turbomachinery*, Vol. 113, Jan. 1991, pp. 42–51.
- Han, J. C., and Zhang, Y. M., and Lee, C. P., "Influence of Surface Heating Condition on Local Heat Transfer in a Rotating Square Channel with Smooth Walls and Radial Outward Flow," *Journal of Turbomachinery*, Vol. 116, No. 1, 1994, pp. 149–158.
- Guidez, J., "Study of the Convective Heat Transfer in a Rotating Coolant Channel," *Journal of Turbomachinery*, Vol. 111, Jan. 1989, pp. 43–50.
- Dutta, S., Andrews, M. J., and Han, J. C., "Prediction of Turbulent Heat Transfer in Rotating Smooth Square Ducts," *International Journal of Heat and Mass Transfer*, Vol. 39, No. 4, 1995, pp. 707–715.

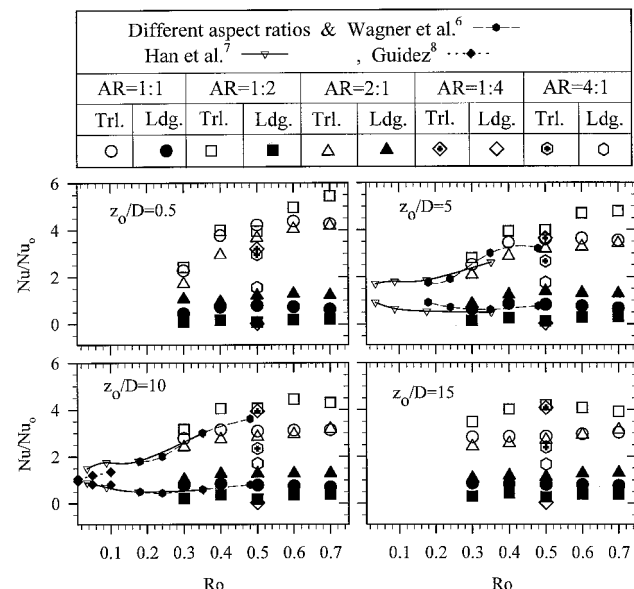


Fig. 2 Effect of rotation number on Nusselt number ratio at selected axial locations.